Section 2.2: Considering Categorical Data

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STAT 1201
Introduction to Probability and Statistics

2.2 Considering categorical data

In this section, we will introduce tables and other basic tools for categorical data that are used throughout this book. The `loan50` data set represents a sample from a larger loan data set called `loans`. This larger data set contains information on 10,000 loans made through Lending Club. We will examine the relationship between `homeownership`, which for the `loans` data can take a value of `rent`, `mortgage` (owns but has a mortgage), or `own`, and `app_type`, which indicates whether the loan application was made with a partner or whether it was an individual application.

2.2.1 Contingency tables and bar plots

Figure 2.17 summarizes two variables: `app_type` and `homeownership`. A table that summarizes data for two categorical variables in this way is called a **contingency table**. Each value in the table represents the number of times a particular combination of variable outcomes occurred. For example, the value 3496 corresponds to the number of loans in the data set where the borrower rents their home and the application type was by an individual. Row and column totals are also included. The **row totals** provide the total counts across each row (e.g. $3496 + 3839 + 1170 = 8505$), and **column totals** are total counts down each column. We can also create a table that shows only the overall percentages or proportions for each combination of categories, or we can create a table for a single variable, such as the one shown in Figure 2.18 for the `homeownership` variable.

		`homeownership`			
		rent	mortgage	own	Total
`app_type`	individual	3496	3839	1170	8505
	joint	362	950	183	1495
	Total	3858	4789	1353	10000

Figure 2.17: A contingency table for `app_type` and `homeownership`.

`homeownership`	Count
rent	3858
mortgage	4789
own	1353
Total	10000

Figure 2.18: A table summarizing the frequencies of each value for the `homeownership` variable.

A bar plot is a common way to display a single categorical variable. The left panel of Figure 2.19 shows a **bar plot** for the `homeownership` variable. In the right panel, the counts are converted into proportions, showing the proportion of observations that are in each level (e.g. $3858/10000 = 0.3858$ for `rent`).

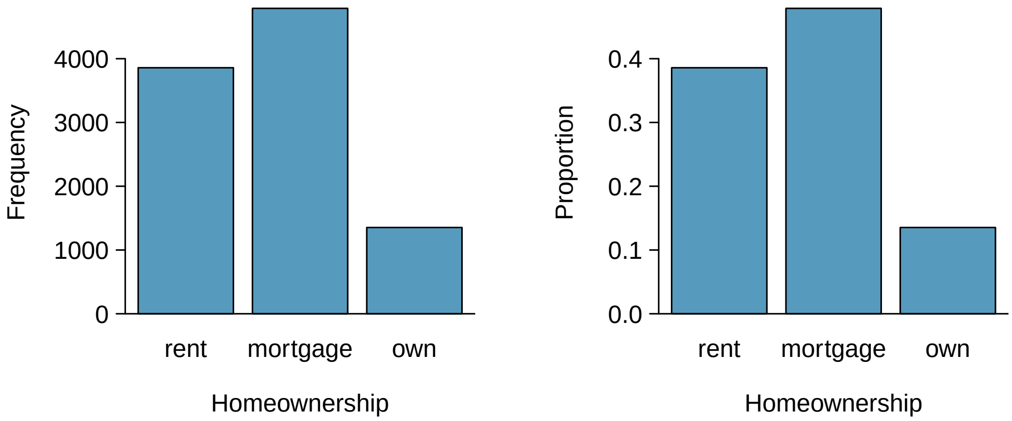

Figure 2.19: Two bar plots of `number`. The left panel shows the counts, and the right panel shows the proportions in each group.

2.2.2 Row and column proportions

Sometimes it is useful to understand the fractional breakdown of one variable in another, and we can modify our contingency table to provide such a view. Figure 2.20 shows the **row proportions** for Figure 2.17, which are computed as the counts divided by their row totals. The value 3496 at the intersection of `individual` and `rent` is replaced by 3496/8505 = 0.411, i.e. 3496 divided by its row total, 8505. So what does 0.411 represent? It corresponds to the proportion of individual applicants who rent.

	rent	mortgage	own	Total
individual	0.411	0.451	0.138	1.000
joint	0.242	0.635	0.122	1.000
Total	0.386	0.479	0.135	1.000

Figure 2.20: A contingency table with row proportions for the `app_type` and `homeownership` variables. The row total is off by 0.001 for the `joint` row due to a rounding error.

A contingency table of the column proportions is computed in a similar way, where each **column proportion** is computed as the count divided by the corresponding column total. Figure 2.21 shows such a table, and here the value 0.906 indicates that 90.6% of renters applied as individuals for the loan. This rate is higher compared to loans from people with mortgages (80.2%) or who own their home (86.5%). Because these rates vary between the three levels of `homeownership` (`rent`, `mortgage`, `own`), this provides evidence that the `app_type` and `homeownership` variables are associated.

	rent	mortgage	own	Total
individual	0.906	0.802	0.865	0.851
joint	0.094	0.198	0.135	0.150
Total	1.000	1.000	1.000	1.000

Figure 2.21: A contingency table with column proportions for the `app_type` and `homeownership` variables. The total for the last column is off by 0.001 due to a rounding error.

We could also have checked for an association between `app_type` and `homeownership` in Figure 2.20 using row proportions. When comparing these row proportions, we would look down columns to see if the fraction of loans where the borrower rents, has a mortgage, or owns varied across the `individual` to `joint` application types.

GUIDED PRACTICE 2.23

G

(a) What does 0.451 represent in Figure 2.20?
(b) What does 0.802 represent in Figure 2.21?[18]

GUIDED PRACTICE 2.24

G

(a) What does 0.122 at the intersection of `joint` and `own` represent in Figure 2.20?
(b) What does 0.135 represent in the Figure 2.21?[19]

EXAMPLE 2.25

E

Data scientists use statistics to filter spam from incoming email messages. By noting specific characteristics of an email, a data scientist may be able to classify some emails as spam or not spam with high accuracy. One such characteristic is whether the email contains no numbers, small numbers, or big numbers. Another characteristic is the email format, which indicates whether or not an email has any HTML content, such as bolded text. We'll focus on email format and spam status using the `email` data set, and these variables are summarized in a contingency table in Figure 2.22. Which would be more helpful to someone hoping to classify email as spam or regular email for this table: row or column proportions?

A data scientist would be interested in how the proportion of spam changes within each email format. This corresponds to column proportions: the proportion of spam in plain text emails and the proportion of spam in HTML emails.

If we generate the column proportions, we can see that a higher fraction of plain text emails are spam (209/1195 = 17.5%) than compared to HTML emails (158/2726 = 5.8%). This information on its own is insufficient to classify an email as spam or not spam, as over 80% of plain text emails are not spam. Yet, when we carefully combine this information with many other characteristics, we stand a reasonable chance of being able to classify some emails as spam or not spam with confidence.

	text	HTML	Total
spam	209	158	367
not spam	986	2568	3554
Total	1195	2726	3921

Figure 2.22: A contingency table for `spam` and `format`.

Example 2.25 points out that row and column proportions are not equivalent. Before settling on one form for a table, it is important to consider each to ensure that the most useful table is constructed. However, sometimes it simply isn't clear which, if either, is more useful.

EXAMPLE 2.26

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Look back to Tables 2.20 and 2.21. Are there any obvious scenarios where one might be more useful than the other?

None that we thought were obvious! What is distinct about `app_type` and `homeownership` vs the email example is that these two variables don't have a clear explanatory-response variable relationship that we might hypothesize (see Section 1.2.4 for these terms). Usually it is most useful to "condition" on the explanatory variable. For instance, in the email example, the email format was seen as a possible explanatory variable of whether the message was spam, so we would find it more interesting to compute the relative frequencies (proportions) for each email format.

[18] (a) 0.451 represents the proportion of individual applicants who have a mortgage. (b) 0.802 represents the fraction of applicants with mortgages who applied as individuals.

[19] (a) 0.122 represents the fraction of joint borrowers who own their home. (b) 0.135 represents the home-owning borrowers who had a joint application for the loan.

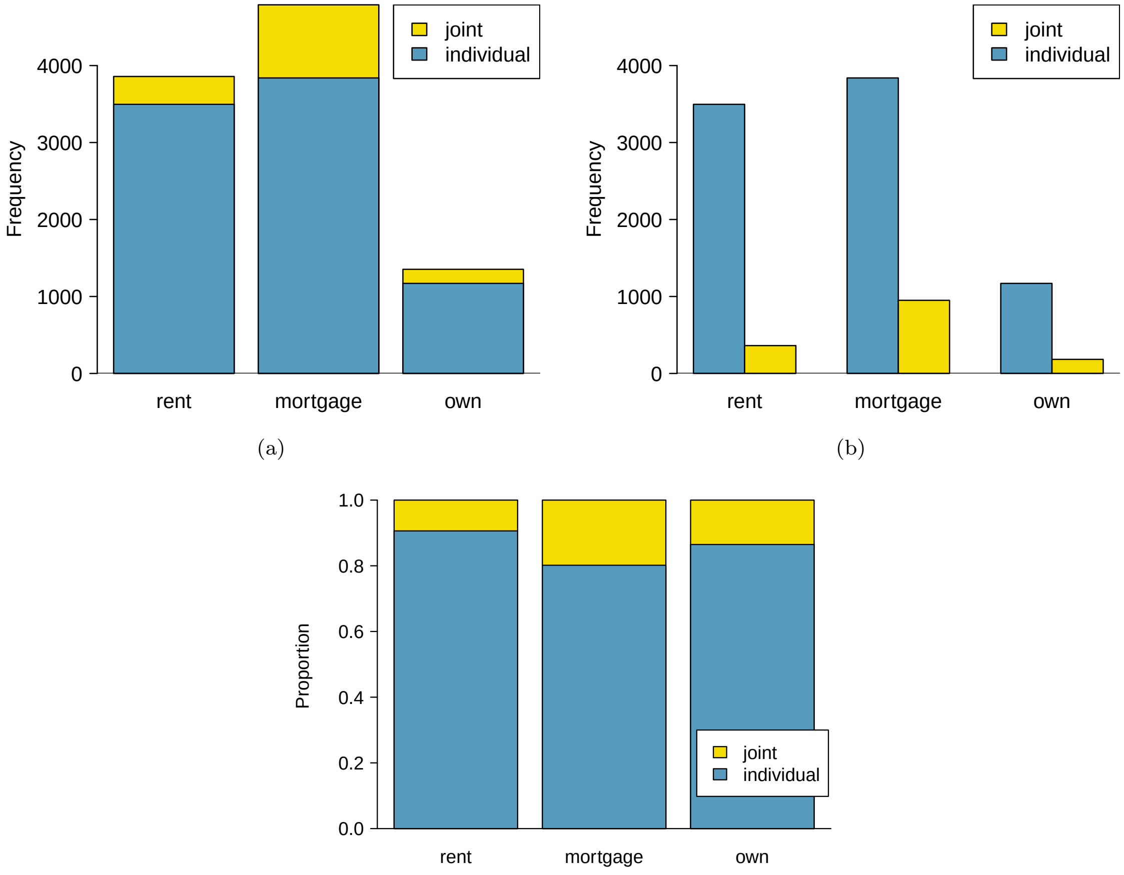
2.2.3 Using a bar plot with two variables

Contingency tables using row or column proportions are especially useful for examining how two categorical variables are related. Stacked bar plots provide a way to visualize the information in these tables.

A **stacked bar plot** is a graphical display of contingency table information. For example, a stacked bar plot representing Figure 2.21 is shown in Figure 2.23(a), where we have first created a bar plot using the `homeownership` variable and then divided each group by the levels of `app_type`.

One related visualization to the stacked bar plot is the **side-by-side bar plot**, where an example is shown in Figure 2.23(b).

For the last type of bar plot we introduce, the column proportions for the `app_type` and `homeownership` contingency table have been translated into a standardized stacked bar plot in Figure 2.23(c). This type of visualization is helpful in understanding the fraction of individual or joint loan applications for borrowers in each level of `homeownership`. Additionally, since the proportions of `joint` and `individual` vary across the groups, we can conclude that the two variables are associated.



(c)

Figure 2.23: (a) Stacked bar plot for `homeownership`, where the counts have been further broken down by `app_type`. (b) Side-by-side bar plot for `homeownership` and `app_type`. (c) Standardized version of the stacked bar plot.

EXAMPLE 2.27

Examine the three bar plots in Figure 2.23. When is the stacked, side-by-side, or standardized stacked bar plot the most useful?

The stacked bar plot is most useful when it's reasonable to assign one variable as the explanatory variable and the other variable as the response, since we are effectively grouping by one variable first and then breaking it down by the others.

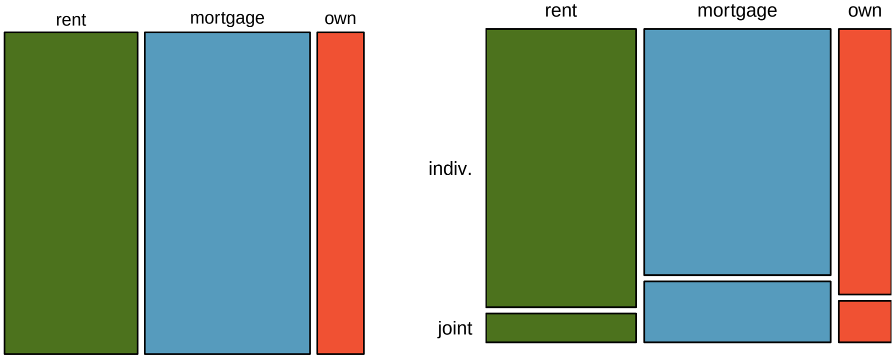
Side-by-side bar plots are more agnostic in their display about which variable, if any, represents the explanatory and which the response variable. It is also easy to discern the number of cases in of the six different group combinations. However, one downside is that it tends to require more horizontal space; the narrowness of Figure 2.23(b) makes the plot feel a bit cramped. Additionally, when two groups are of very different sizes, as we see in the own group relative to either of the other two groups, it is difficult to discern if there is an association between the variables.

The standardized stacked bar plot is helpful if the primary variable in the stacked bar plot is relatively imbalanced, e.g. the own category has only a third of the observations in the mortgage category, making the simple stacked bar plot less useful for checking for an association. The major downside of the standardized version is that we lose all sense of how many cases each of the bars represents.

2.2.4 Mosaic plots

A **mosaic plot** is a visualization technique suitable for contingency tables that resembles a standardized stacked bar plot with the benefit that we still see the relative group sizes of the primary variable as well.

To get started in creating our first mosaic plot, we'll break a square into columns for each category of the homeownership variable, with the result shown in Figure 2.24(a). Each column represents a level of homeownership, and the column widths correspond to the proportion of loans in each of those categories. For instance, there are fewer loans where the borrower is an owner than where the borrower has a mortgage. In general, mosaic plots use box *areas* to represent the number of cases in each category.



(a) (b)

Figure 2.24: (a) The one-variable mosaic plot for homeownership. (b) Two-variable mosaic plot for both homeownership and app_type.

To create a completed mosaic plot, the single-variable mosaic plot is further divided into pieces in Figure 2.24(b) using the app_type variable. Each column is split proportional to the number of loans from individual and joint borrowers. For example, the second column represents loans where the borrower has a mortgage, and it was divided into individual loans (upper) and joint loans (lower). As another example, the bottom segment of the third column represents loans where the borrower owns their home and applied jointly, while the upper segment of this column represents borrowers who are homeowners and filed individually. We can again use this plot to see that the homeownership and app_type variables are associated, since some columns are divided in different vertical locations than others, which was the same technique used for checking an association in the standardized stacked bar plot.

In Figure 2.24, we chose to first split by the homeowner status of the borrower. However, we could have instead first split by the application type, as in Figure 2.25. Like with the bar plots, it's common to use the explanatory variable to represent the first split in a mosaic plot, and then for the response to break up each level of the explanatory variable, if these labels are reasonable to attach to the variables under consideration.

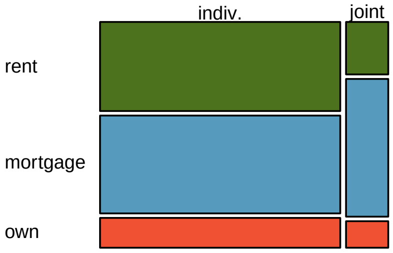


Figure 2.25: Mosaic plot where loans are grouped by the `homeownership` variable after they've been divided into the `individual` and `joint` application types.

2.2.5 The only pie chart you will see in this book

A **pie chart** is shown in Figure 2.26 alongside a bar plot representing the same information. Pie charts can be useful for giving a high-level overview to show how a set of cases break down. However, it is also difficult to decipher details in a pie chart. For example, it takes a couple seconds longer to recognize that there are more loans where the borrower has a mortgage than rent when looking at the pie chart, while this detail is very obvious in the bar plot. While pie charts can be useful, we prefer bar plots for their ease in comparing groups.

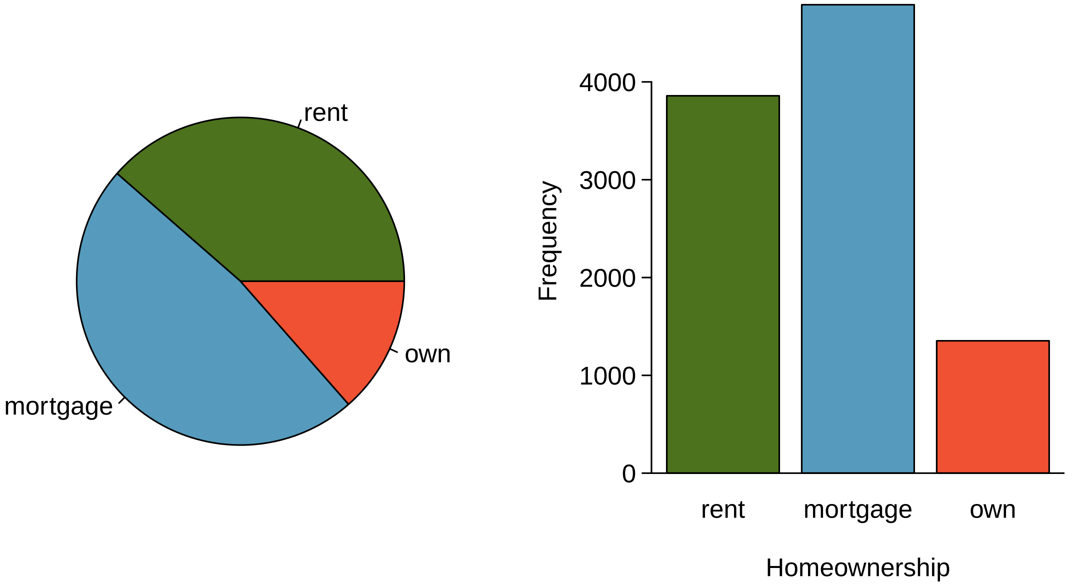


Figure 2.26: A pie chart and bar plot of `homeownership`.

2.2.6 Comparing numerical data across groups

Some of the more interesting investigations can be considered by examining numerical data across groups. The methods required here aren't really new: all that's required is to make a numerical plot for each group in the same graph. Here two convenient methods are introduced: side-by-side box plots and hollow histograms.

We will take a look again at the `county` data set and compare the median household income for counties that gained population from 2010 to 2017 versus counties that had no gain. While we might like to make a causal connection here, remember that these are observational data and so such an interpretation would be, at best, half-baked.

There were 1,454 counties where the population increased from 2010 to 2017, and there were 1,672 counties with no gain (all but one were a loss). A random sample of 100 counties from the first group and 50 from the second group are shown in Figure 2.27 to give a better sense of some of the raw median income data.

Median Income for 150 Counties, in $1000s

Population Gain						No Population Gain		
38.2	43.6	42.2	61.5	51.1	45.7	48.3	60.3	50.7
44.6	51.8	40.7	48.1	56.4	41.9	39.3	40.4	40.3
40.6	63.3	52.1	60.3	49.8	51.7	57	47.2	45.9
51.1	34.1	45.5	52.8	49.1	51	42.3	41.5	46.1
80.8	46.3	82.2	43.6	39.7	49.4	44.9	51.7	46.4
75.2	40.6	46.3	62.4	44.1	51.3	29.1	51.8	50.5
51.9	34.7	54	42.9	52.2	45.1	27	30.9	34.9
61	51.4	56.5	62	46	46.4	40.7	51.8	61.1
53.8	57.6	69.2	48.4	40.5	48.6	43.4	34.7	45.7
53.1	54.6	55	46.4	39.9	56.7	33.1	21	37
63	49.1	57.2	44.1	50	38.9	52	31.9	45.7
46.6	46.5	38.9	50.9	56	34.6	56.3	38.7	45.7
74.2	63	49.6	53.7	77.5	60	56.2	43	21.7
63.2	47.6	55.9	39.1	57.8	42.6	44.5	34.5	48.9
50.4	49	45.6	39	38.8	37.1	50.9	42.1	43.2
57.2	44.7	71.7	35.3	100.2		35.4	41.3	33.6
42.6	55.5	38.6	52.7	63		43.4	56.5	

Figure 2.27: In this table, median household income (in $1000s) from a random sample of 100 counties that had population gains are shown on the left. Median incomes from a random sample of 50 counties that had no population gain are shown on the right.

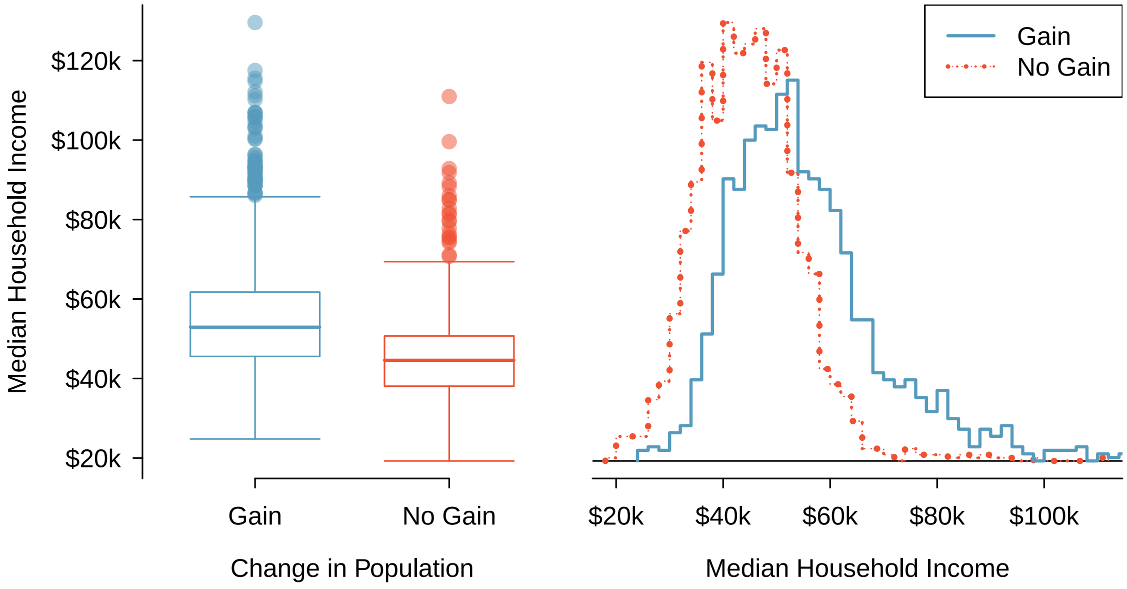


Figure 2.28: Side-by-side box plot (left panel) and hollow histograms (right panel) for `med_hh_income`, where the counties are split by whether there was a population gain or loss.

The **side-by-side box plot** is a traditional tool for comparing across groups. An example is shown in the left panel of Figure 2.28, where there are two box plots, one for each group, placed into one plotting window and drawn on the same scale.

Another useful plotting method uses **hollow histograms** to compare numerical data across groups. These are just the outlines of histograms of each group put on the same plot, as shown in the right panel of Figure 2.28.

G

GUIDED PRACTICE 2.28

Use the plots in Figure 2.28 to compare the incomes for counties across the two groups. What do you notice about the approximate center of each group? What do you notice about the variability between groups? Is the shape relatively consistent between groups? How many *prominent* modes are there for each group?[20]

G

GUIDED PRACTICE 2.29

What components of each plot in Figure 2.28 do you find most useful?[21]

[20] Answers may vary a little. The counties with population gains tend to have higher income (median of about $45,000) versus counties without a gain (median of about $40,000). The variability is also slightly larger for the population gain group. This is evident in the IQR, which is about 50% bigger in the *gain* group. Both distributions show slight to moderate right skew and are unimodal. The box plots indicate there are many observations far above the median in each group, though we should anticipate that many observations will fall beyond the whiskers when examining any data set that contain more than a couple hundred data points.

[21] Answers will vary. The side-by-side box plots are especially useful for comparing centers and spreads, while the hollow histograms are more useful for seeing distribution shape, skew, and potential anomalies.