



Practice Exercises: Lesson 2.1

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STAT 1201
Introduction to Probability and Statistics

ONLINE AND DISTANCE EDUCATION

Exercises

3.1 True or false. Determine if the statements below are true or false, and explain your reasoning.

- If a fair coin is tossed many times and the last eight tosses are all heads, then the chance that the next toss will be heads is somewhat less than 50%.
- Drawing a face card (jack, queen, or king) and drawing a red card from a full deck of playing cards are mutually exclusive events.
- Drawing a face card and drawing an ace from a full deck of playing cards are mutually exclusive events.

3.2 Roulette wheel. The game of roulette involves spinning a wheel with 38 slots: 18 red, 18 black, and 2 green. A ball is spun onto the wheel and will eventually land in a slot, where each slot has an equal chance of capturing the ball.

- You watch a roulette wheel spin 3 consecutive times and the ball lands on a red slot each time. What is the probability that the ball will land on a red slot on the next spin?
- You watch a roulette wheel spin 300 consecutive times and the ball lands on a red slot each time. What is the probability that the ball will land on a red slot on the next spin?
- Are you equally confident of your answers to parts (a) and (b)? Why or why not?



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3.3 Four games, one winner. Below are four versions of the same game. Your archnemesis gets to pick the version of the game, and then you get to choose how many times to flip a coin: 10 times or 100 times. Identify how many coin flips you should choose for each version of the game. It costs \$1 to play each game. Explain your reasoning.

- If the proportion of heads is larger than 0.60, you win \$1.
- If the proportion of heads is larger than 0.40, you win \$1.
- If the proportion of heads is between 0.40 and 0.60, you win \$1.
- If the proportion of heads is smaller than 0.30, you win \$1.

3.4 Backgammon. Backgammon is a board game for two players in which the playing pieces are moved according to the roll of two dice. Players win by removing all of their pieces from the board, so it is usually good to roll high numbers. You are playing backgammon with a friend and you roll two 6s in your first roll and two 6s in your second roll. Your friend rolls two 3s in his first roll and again in his second row. Your friend claims that you are cheating, because rolling double 6s twice in a row is very unlikely. Using probability, show that your rolls were just as likely as his.

3.5 Coin flips. If you flip a fair coin 10 times, what is the probability of

- getting all tails?
- getting all heads?
- getting at least one tails?

3.6 Dice rolls. If you roll a pair of fair dice, what is the probability of

- getting a sum of 1?
- getting a sum of 5?
- getting a sum of 12?

3.7 Swing voters. A Pew Research survey asked 2,373 randomly sampled registered voters their political affiliation (Republican, Democrat, or Independent) and whether or not they identify as swing voters. 35% of respondents identified as Independent, 23% identified as swing voters, and 11% identified as both.²¹

- Are being Independent and being a swing voter disjoint, i.e. mutually exclusive?
- Draw a Venn diagram summarizing the variables and their associated probabilities.
- What percent of voters are Independent but not swing voters?
- What percent of voters are Independent or swing voters?
- What percent of voters are neither Independent nor swing voters?
- Is the event that someone is a swing voter independent of the event that someone is a political Independent?

3.8 Poverty and language. The American Community Survey is an ongoing survey that provides data every year to give communities the current information they need to plan investments and services. The 2010 American Community Survey estimates that 14.6% of Americans live below the poverty line, 20.7% speak a language other than English (foreign language) at home, and 4.2% fall into both categories.²²

- Are living below the poverty line and speaking a foreign language at home disjoint?
- Draw a Venn diagram summarizing the variables and their associated probabilities.
- What percent of Americans live below the poverty line and only speak English at home?
- What percent of Americans live below the poverty line or speak a foreign language at home?
- What percent of Americans live above the poverty line and only speak English at home?
- Is the event that someone lives below the poverty line independent of the event that the person speaks a foreign language at home?

3.9 Disjoint vs. independent. In parts (a) and (b), identify whether the events are disjoint, independent, or neither (events cannot be both disjoint and independent).

- You and a randomly selected student from your class both earn A's in this course.
- You and your class study partner both earn A's in this course.
- If two events can occur at the same time, must they be dependent?

3.10 Guessing on an exam. In a multiple choice exam, there are 5 questions and 4 choices for each question (a, b, c, d). Nancy has not studied for the exam at all and decides to randomly guess the answers. What is the probability that:

- the first question she gets right is the 5th question?
- she gets all of the questions right?
- she gets at least one question right?

²¹Pew Research Center, With Voters Focused on Economy, Obama Lead Narrows, data collected between April 4-15, 2012.

²²U.S. Census Bureau, 2010 American Community Survey 1-Year Estimates, Characteristics of People by Language Spoken at Home.

3.11 Educational attainment of couples. The table below shows the distribution of education level attained by US residents by gender based on data collected in the 2010 American Community Survey.²³

| | | <i>Gender</i> | |
|---|---------------------------------|---------------|--------|
| | | Male | Female |
| <i>Highest education attained</i> | Less than 9th grade | 0.07 | 0.13 |
| | 9th to 12th grade, no diploma | 0.10 | 0.09 |
| | HS graduate (or equivalent) | 0.30 | 0.20 |
| | Some college, no degree | 0.22 | 0.24 |
| | Associate's degree | 0.06 | 0.08 |
| | Bachelor's degree | 0.16 | 0.17 |
| | Graduate or professional degree | 0.09 | 0.09 |
| Total | | 1.00 | 1.00 |

- What is the probability that a randomly chosen man has at least a Bachelor's degree?
- What is the probability that a randomly chosen woman has at least a Bachelor's degree?
- What is the probability that a man and a woman getting married both have at least a Bachelor's degree? Note any assumptions you must make to answer this question.
- If you made an assumption in part (c), do you think it was reasonable? If you didn't make an assumption, double check your earlier answer and then return to this part.

3.12 School absences. Data collected at elementary schools in DeKalb County, GA suggest that each year roughly 25% of students miss exactly one day of school, 15% miss 2 days, and 28% miss 3 or more days due to sickness.²⁴

- What is the probability that a student chosen at random doesn't miss any days of school due to sickness this year?
- What is the probability that a student chosen at random misses no more than one day?
- What is the probability that a student chosen at random misses at least one day?
- If a parent has two kids at a DeKalb County elementary school, what is the probability that neither kid will miss any school? Note any assumption you must make to answer this question.
- If a parent has two kids at a DeKalb County elementary school, what is the probability that both kids will miss some school, i.e. at least one day? Note any assumption you make.
- If you made an assumption in part (d) or (e), do you think it was reasonable? If you didn't make any assumptions, double check your earlier answers.

²³U.S. Census Bureau, 2010 American Community Survey 1-Year Estimates, Educational Attainment.

²⁴S.S. Mizan et al. "Absence, Extended Absence, and Repeat Tardiness Related to Asthma Status among Elementary School Children". In: *Journal of Asthma* 48.3 (2011), pp. 228–234.

Chapter exercises

3.39 Grade distributions. Each row in the table below is a proposed grade distribution for a class. Identify each as a valid or invalid probability distribution, and explain your reasoning.

| | <i>Grades</i> | | | | |
|-----|---------------|------|-----|-----|------|
| | A | B | C | D | F |
| (a) | 0.3 | 0.3 | 0.3 | 0.2 | 0.1 |
| (b) | 0 | 0 | 1 | 0 | 0 |
| (c) | 0.3 | 0.3 | 0.3 | 0 | 0 |
| (d) | 0.3 | 0.5 | 0.2 | 0.1 | -0.1 |
| (e) | 0.2 | 0.4 | 0.2 | 0.1 | 0.1 |
| (f) | 0 | -0.1 | 1.1 | 0 | 0 |

3.40 Health coverage, frequencies. The Behavioral Risk Factor Surveillance System (BRFSS) is an annual telephone survey designed to identify risk factors in the adult population and report emerging health trends. The following table summarizes two variables for the respondents: health status and health coverage, which describes whether each respondent had health insurance.⁶⁴

| | | <i>Health Status</i> | | | | | |
|------------------------|-------|----------------------|-----------|-------|-------|------|--------|
| | | Excellent | Very good | Good | Fair | Poor | Total |
| <i>Health Coverage</i> | No | 459 | 727 | 854 | 385 | 99 | 2,524 |
| | Yes | 4,198 | 6,245 | 4,821 | 1,634 | 578 | 17,476 |
| | Total | 4,657 | 6,972 | 5,675 | 2,019 | 677 | 20,000 |

- (a) If we draw one individual at random, what is the probability that the respondent has excellent health and doesn't have health coverage?
- (b) If we draw one individual at random, what is the probability that the respondent has excellent health or doesn't have health coverage?

3.41 HIV in Swaziland. Swaziland has the highest HIV prevalence in the world: 25.9% of this country's population is infected with HIV.⁶⁵ The ELISA test is one of the first and most accurate tests for HIV. For those who carry HIV, the ELISA test is 99.7% accurate. For those who do not carry HIV, the test is 92.6% accurate. If an individual from Swaziland has tested positive, what is the probability that he carries HIV?

3.42 Twins. About 30% of human twins are identical, and the rest are fraternal. Identical twins are necessarily the same sex – half are males and the other half are females. One-quarter of fraternal twins are both male, one-quarter both female, and one-half are mixes: one male, one female. You have just become a parent of twins and are told they are both girls. Given this information, what is the probability that they are identical?

3.43 Cost of breakfast. Sally gets a cup of coffee and a muffin every day for breakfast from one of the many coffee shops in her neighborhood. She picks a coffee shop each morning at random and independently of previous days. The average price of a cup of coffee is \$1.40 with a standard deviation of 30¢ (\$0.30), the average price of a muffin is \$2.50 with a standard deviation of 15¢, and the two prices are independent of each other.

- (a) What is the mean and standard deviation of the amount she spends on breakfast daily?
- (b) What is the mean and standard deviation of the amount she spends on breakfast weekly (7 days)?

⁶⁴Office of Surveillance, Epidemiology, and Laboratory Services Behavioral Risk Factor Surveillance System, BRFSS 2010 Survey Data.

⁶⁵Source: CIA Factbook, Country Comparison: HIV/AIDS - Adult Prevalence Rate.