



Practice Exercises: Lesson 3.2

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STAT 1201
Introduction to Probability and Statistics

ONLINE AND DISTANCE EDUCATION

Exercises

5.7 Chronic illness, Part I. In 2013, the Pew Research Foundation reported that “45% of U.S. adults report that they live with one or more chronic conditions”.¹¹ However, this value was based on a sample, so it may not be a perfect estimate for the population parameter of interest on its own. The study reported a standard error of about 1.2%, and a normal model may reasonably be used in this setting. Create a 95% confidence interval for the proportion of U.S. adults who live with one or more chronic conditions. Also interpret the confidence interval in the context of the study.

5.8 Twitter users and news, Part I. A poll conducted in 2013 found that 52% of U.S. adult Twitter users get at least some news on Twitter.¹² The standard error for this estimate was 2.4%, and a normal distribution may be used to model the sample proportion. Construct a 99% confidence interval for the fraction of U.S. adult Twitter users who get some news on Twitter, and interpret the confidence interval in context.

5.9 Chronic illness, Part II. In 2013, the Pew Research Foundation reported that “45% of U.S. adults report that they live with one or more chronic conditions”, and the standard error for this estimate is 1.2%. Identify each of the following statements as true or false. Provide an explanation to justify each of your answers.

- We can say with certainty that the confidence interval from Exercise 5.7 contains the true percentage of U.S. adults who suffer from a chronic illness.
- If we repeated this study 1,000 times and constructed a 95% confidence interval for each study, then approximately 950 of those confidence intervals would contain the true fraction of U.S. adults who suffer from chronic illnesses.
- The poll provides statistically significant evidence (at the $\alpha = 0.05$ level) that the percentage of U.S. adults who suffer from chronic illnesses is below 50%.
- Since the standard error is 1.2%, only 1.2% of people in the study communicated uncertainty about their answer.

5.10 Twitter users and news, Part II. A poll conducted in 2013 found that 52% of U.S. adult Twitter users get at least some news on Twitter, and the standard error for this estimate was 2.4%. Identify each of the following statements as true or false. Provide an explanation to justify each of your answers.

- The data provide statistically significant evidence that more than half of U.S. adult Twitter users get some news through Twitter. Use a significance level of $\alpha = 0.01$. (This part uses concepts from Section 5.3 and will be corrected in a future edition.)
- Since the standard error is 2.4%, we can conclude that 97.6% of all U.S. adult Twitter users were included in the study.
- If we want to reduce the standard error of the estimate, we should collect less data.
- If we construct a 90% confidence interval for the percentage of U.S. adults Twitter users who get some news through Twitter, this confidence interval will be wider than a corresponding 99% confidence interval.

¹¹Pew Research Center, Washington, D.C. The Diagnosis Difference, November 26, 2013.

¹²Pew Research Center, Washington, D.C. Twitter News Consumers: Young, Mobile and Educated, November 4, 2013.

5.11 Waiting at an ER, Part I. A hospital administrator hoping to improve wait times decides to estimate the average emergency room waiting time at her hospital. She collects a simple random sample of 64 patients and determines the time (in minutes) between when they checked in to the ER until they were first seen by a doctor. A 95% confidence interval based on this sample is (128 minutes, 147 minutes), which is based on the normal model for the mean. Determine whether the following statements are true or false, and explain your reasoning.

- (a) We are 95% confident that the average waiting time of these 64 emergency room patients is between 128 and 147 minutes.
- (b) We are 95% confident that the average waiting time of all patients at this hospital's emergency room is between 128 and 147 minutes.
- (c) 95% of random samples have a sample mean between 128 and 147 minutes.
- (d) A 99% confidence interval would be narrower than the 95% confidence interval since we need to be more sure of our estimate.
- (e) The margin of error is 9.5 and the sample mean is 137.5.
- (f) In order to decrease the margin of error of a 95% confidence interval to half of what it is now, we would need to double the sample size. (Hint: the margin of error for a mean scales in the same way with sample size as the margin of error for a proportion.)

5.12 Mental health. The General Social Survey asked the question: "For how many days during the past 30 days was your mental health, which includes stress, depression, and problems with emotions, not good?" Based on responses from 1,151 US residents, the survey reported a 95% confidence interval of 3.40 to 4.24 days in 2010.

- (a) Interpret this interval in context of the data.
- (b) What does "95% confident" mean? Explain in the context of the application.
- (c) Suppose the researchers think a 99% confidence level would be more appropriate for this interval. Will this new interval be smaller or wider than the 95% confidence interval?
- (d) If a new survey were to be done with 500 Americans, do you think the standard error of the estimate be larger, smaller, or about the same.

5.13 Website registration. A website is trying to increase registration for first-time visitors, exposing 1% of these visitors to a new site design. Of 752 randomly sampled visitors over a month who saw the new design, 64 registered.

- (a) Check any conditions required for constructing a confidence interval.
- (b) Compute the standard error.
- (c) Construct and interpret a 90% confidence interval for the fraction of first-time visitors of the site who would register under the new design (assuming stable behaviors by new visitors over time).

5.14 Coupons driving visits. A store randomly samples 603 shoppers over the course of a year and finds that 142 of them made their visit because of a coupon they'd received in the mail. Construct a 95% confidence interval for the fraction of all shoppers during the year whose visit was because of a coupon they'd received in the mail.

Chapter exercises

5.27 Relaxing after work. The General Social Survey asked the question: “After an average work day, about how many hours do you have to relax or pursue activities that you enjoy?” to a random sample of 1,155 Americans.²⁵ A 95% confidence interval for the mean number of hours spent relaxing or pursuing activities they enjoy was (1.38, 1.92).

- Interpret this interval in context of the data.
- Suppose another set of researchers reported a confidence interval with a larger margin of error based on the same sample of 1,155 Americans. How does their confidence level compare to the confidence level of the interval stated above?
- Suppose next year a new survey asking the same question is conducted, and this time the sample size is 2,500. Assuming that the population characteristics, with respect to how much time people spend relaxing after work, have not changed much within a year. How will the margin of error of the 95% confidence interval constructed based on data from the new survey compare to the margin of error of the interval stated above?

5.28 Minimum wage, Part II. In Exercise 5.21, we learned that a Rasmussen Reports survey of 1,000 US adults found that 42% believe raising the minimum wage will help the economy. Construct a 99% confidence interval for the true proportion of US adults who believe this.

5.29 Testing for food safety. A food safety inspector is called upon to investigate a restaurant with a few customer reports of poor sanitation practices. The food safety inspector uses a hypothesis testing framework to evaluate whether regulations are not being met. If he decides the restaurant is in gross violation, its license to serve food will be revoked.

- Write the hypotheses in words.
- What is a Type 1 Error in this context?
- What is a Type 2 Error in this context?
- Which error is more problematic for the restaurant owner? Why?
- Which error is more problematic for the diners? Why?
- As a diner, would you prefer that the food safety inspector requires strong evidence or very strong evidence of health concerns before revoking a restaurant’s license? Explain your reasoning.

5.30 True or false. Determine if the following statements are true or false, and explain your reasoning. If false, state how it could be corrected.

- If a given value (for example, the null hypothesized value of a parameter) is within a 95% confidence interval, it will also be within a 99% confidence interval.
- Decreasing the significance level (α) will increase the probability of making a Type 1 Error.
- Suppose the null hypothesis is $p = 0.5$ and we fail to reject H_0 . Under this scenario, the true population proportion is 0.5.
- With large sample sizes, even small differences between the null value and the observed point estimate, a difference often called the effect size, will be identified as statistically significant.

5.31 Unemployment and relationship problems. A USA Today/Gallup poll asked a group of unemployed and underemployed Americans if they have had major problems in their relationships with their spouse or another close family member as a result of not having a job (if unemployed) or not having a full-time job (if underemployed). 27% of the 1,145 unemployed respondents and 25% of the 675 underemployed respondents said they had major problems in relationships as a result of their employment status.

- What are the hypotheses for evaluating if the proportions of unemployed and underemployed people who had relationship problems were different?
- The p-value for this hypothesis test is approximately 0.35. Explain what this means in context of the hypothesis test and the data.

²⁵National Opinion Research Center, General Social Survey, 2018.

5.32 Nearsighted. It is believed that nearsightedness affects about 8% of all children. In a random sample of 194 children, 21 are nearsighted. Conduct a hypothesis test for the following question: do these data provide evidence that the 8% value is inaccurate?

5.33 Nutrition labels. The nutrition label on a bag of potato chips says that a one ounce (28 gram) serving of potato chips has 130 calories and contains ten grams of fat, with three grams of saturated fat. A random sample of 35 bags yielded a confidence interval for the number of calories per bag of 128.2 to 139.8 calories. Is there evidence that the nutrition label does not provide an accurate measure of calories in the bags of potato chips?

5.34 CLT for proportions. Define the term “sampling distribution” of the sample proportion, and describe how the shape, center, and spread of the sampling distribution change as the sample size increases when $p = 0.1$.

5.35 Practical vs. statistical significance. Determine whether the following statement is true or false, and explain your reasoning: “With large sample sizes, even small differences between the null value and the observed point estimate can be statistically significant.”

5.36 Same observation, different sample size. Suppose you conduct a hypothesis test based on a sample where the sample size is $n = 50$, and arrive at a p-value of 0.08. You then refer back to your notes and discover that you made a careless mistake, the sample size should have been $n = 500$. Will your p-value increase, decrease, or stay the same? Explain.

5.37 Gender pay gap in medicine. A study examined the average pay for men and women entering the workforce as doctors for 21 different positions.²⁶

- (a) If each gender was equally paid, then we would expect about half of those positions to have men paid more than women and women would be paid more than men in the other half of positions. Write appropriate hypotheses to test this scenario.
- (b) Men were, on average, paid more in 19 of those 21 positions. Complete a hypothesis test using your hypotheses from part (a).

²⁶Lo Sasso AT et al. “The \$16,819 Pay Gap For Newly Trained Physicians: The Unexplained Trend Of Men Earning More Than Women”. In: *Health Affairs* 30.2 (2011).